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Robust Adaptive Tracking Control for Time-varying Nonlinear Systems with Higher Order Relative Degree

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Abstract
This paper deals with a controller design problem for time-varying nonlinear systems with nonparametric uncertainties. A robust adaptive tracking control for uncertain time-varying nonlinear systems with higher order relative degree will be proposed based on the high-gain adaptive output feedback and backstepping strategies. The proposed method is useful in the case where only the output signal is available.

1 INTRODUCTION
A nonlinear system is said to be OFEP (output feedback exponentially passive) [1] if there exists an output feedback such that the resulting closed-loop system is exponentially passive. The sufficient conditions for the nonlinear system to be OFEP are that (1) the system be globally exponential minimum-phase, (2) the system has relative degree of 1 and (3) the nonlinearities of the system satisfy the Lipschitz condition. Under these conditions, there exists a static output feedback such that the resulting closed-loop system is exponentially passive[1]. It has also been shown that one can stabilize uncertain nonlinear systems with OFEP property by a high-gain feedback based adaptive control with simple structure [2], [4]. Since the control methods utilize only the output signal in order to design the controller and have strong robustness with respect to bounded disturbances in spite of its simple structure, these methods are considered powerful control tools for uncertain nonlinear systems. Unfortunately the OFEP conditions give very severe restrictions to practical applications of the above-mentioned adaptive schemes because most practical systems do not satisfy the OFEP condition.

With this problem in mind, some alleviation methods to the OFEP condition have been proposed [3], [4], [6]. The method by [3] and [4] alleviated the OFEP condition by introducing a parallel feedforward compensator (PFC) in parallel to the controlled system. Although this method can solve the restriction for relative degree, since the controller is designed for an augmented controlled system with PFC, the bias error from the PFC output may remain. The method by [6] is a robust control scheme for non-OFEP systems with nonlinear uncertainties but the method was for systems with relative degree of 1.

In this paper, we will propose a robust adaptive tracking control, which is based on high-gain feedback based adaptive control, for a class of uncertain time-varying nonlinear systems with higher order relative degree. We extend the robust adaptive control method by [6] to uncertain time-varying nonlinear systems with higher order relative degree by utilizing a backstepping strategy. It is shown that if the upper limit of uncertain nonlinearities can be evaluated by a function of the output signal then one can design an adaptive controller by using only the output signal without a state observer.

2 PROBLEM STATEMENT
We consider the following nth order time-varying nonlinear system with relative degree of n.

\[ \dot{x}_i = f_i(x, t) + g_i(t)x_{i+1} \quad (1 \leq i \leq n-1) \]

\[ \dot{x}_n = f_n(x, t) + g_n(t)u \]

\[ y = x_1 \]

where \( x = [x_1, \ldots, x_n]^T \in \mathbb{R}^n \) is the state variable, \( u \) and \( y \in \mathbb{R} \) are the input and output, respectively. \( f_i(x, t) \) are uncertain nonlinearities and \( g_i(t), (1 \leq i \leq n) \) are unknown time-varying functions. We assume that the uncertainties satisfy the following assumptions.

Assumptions: (A-1) Uncertain nonlinear functions \( f_i(x, t) \) can be evaluated for all \( x \in \mathbb{R}^n \) and \( t \in R^+ \) by

\[ |f_i(x, t)| \leq d_{1i}|\psi_i(y)| + d_{0i} \quad (1 \leq i \leq n) \]

with unknown positive constants \( d_{1i}, d_{0i} \) and a known smooth function \( \psi_i(y) \) which has the following property for any variable \( y_1 \) and \( y_2 \):

\[ |\psi_i(y_1 + y_2)| \leq |\psi_i(y_1) + \psi_i(y_2)| + |\psi_i(y_2)| \]

with a known smooth function \( \psi\cdot(y) \) and a function \( \psi\cdot(y) \) which is bounded for all bounded \( y_2 \).

(A-2) Unknown functions \( g_i(t) \) are smooth and bounded with bounded derivatives for any \( t \geq 0 \), and are positive functions such that \( g_i(t) \geq g_{0i} > 0 \) with positive constants \( g_{0i} \).
The control objective is to achieve the goal:
\[ \lim_{t \to \infty} |y(t) - y^*(t)| \leq \delta \]  
(4)

under the assumptions (A-1) and (A-2) for a given \( \delta \) and a command signal \( y^*(t) \) such that \( |y^*(t)| \leq \beta_0 \), \( |y^*(t)| \leq \beta_1 \) with positive constants \( \beta_0 \) and \( \beta_1 \).

### 3 ADAPTIVE CONTROLLER DESIGN

#### 3.1 Virtual System

In order to design a robust adaptive controller based on the high-gain feedback strategy, we must first consider the virtual controlled system by introducing a virtual control input filter.

Consider the \((n - 1)\)th order virtual filter:
\[
\begin{align*}
    u_{i1} &= -\lambda_i u_{i1} + u_{i1+1} \quad (1 \leq i \leq n - 2) \\
    u_{f1} &= -\lambda_{n-1} u_{f1-1} + u \\
    \lambda_i &> 0, \quad (1 \leq i \leq n - 1)
\end{align*}
\]
(5)

The virtual system, which is obtained by considering \( u_{i1} \) given from a virtual filter as the control input, can be represented by the following form with appropriate variable transformation using filtered signals \( u_{f1} \).

\[
\begin{align*}
    \dot{y}(t) &= a(y, \eta, t) + g_{1,n}(t) u_{f1} + f_1(y, \eta, t) \\
    \dot{\eta}(t) &= q(y, \eta, t) + F(y, \eta, t)
\end{align*}
\]
(6)

where \( y = x_1, \eta = [\eta_2, \ldots, \eta_n] \) and \( \eta_k \) are given as follows:

For \( n = 2 \):
\[ \eta_2 = \tilde{g}_{2,2} - \chi_{2,1} \tilde{g}_{1,2} x_1 - u_{f1} \]  
(7)

For \( n \geq 3 \):
\[ \eta_k = \tilde{g}_{k,n} x_k - \sum_{i=3}^k \chi_{k,i-1} \tilde{g}_{i,k-2} x_{k-i+2} \\
- \chi_{k,k-1} \tilde{g}_{1,k-1} x_1 - u_{f1} \]
\[ \eta_n = \tilde{g}_{n,n} x_n - \sum_{i=1}^{n-2} \chi_{n,i} \tilde{g}_{i,n-i} x_{n-i} \\
- \chi_{n,n-1} \tilde{g}_{1,n-1} x_1 - u_{f1-1} \]  
(8)

with
\[
\begin{align*}
    \chi_{1,1} &= 0, \quad \chi_{i,1} = \lambda_{i-1} + g_{i,n} \tilde{g}_{i,n} \quad (2 \leq i \leq n) \\
    \chi_{n,n} &= -\lambda_{n-1} \chi_{n,n-1} \tilde{g}_{n,n-1} - \chi_{n,n-1} \tilde{g}_{1,n} - \chi_{n,n-1} \tilde{g}_{1,n} \\
    \chi_{i,2} &= \sum_{k=2}^n \chi_{k,1} - \sum_{k=2}^i \chi_{k-1,1} \quad (2 \leq i \leq n - 1, \ n \geq 3) \\
    \chi_{i,i+1} &= -\lambda_{i-1} \chi_{i,i} + \chi_{i+1,n-1} \tilde{g}_{i,n} \\
- \chi_{i+1,i} \tilde{g}_{1,i+1} - \chi_{i,i}, \ (2 \leq i \leq n - 1, \ n \geq 3) \\
    \chi_{n,i} &= -\lambda_{n-1} \chi_{n,i-1} - \chi_{n,i-1} \\
- \chi_{n,i} \tilde{g}_{1,i} - \chi_{n,i}, \ (2 \leq i \leq n - 1, \ n \geq 3) \\
    \chi_{k,i} &= -\lambda_{k-1} \chi_{k,i} - \chi_{k+1,n-i-k} - \chi_{k+1,n-i-k} \\
- \chi_{k+1,i} \tilde{g}_{1,n-i-k} - \chi_{k+1,i} \tilde{g}_{1,n-i-k} \quad (2 \leq i \leq n - 1, \ n \geq 4) \\
    \chi_{k,i} &= -\lambda_{k-1} \chi_{k,i} - \chi_{k+1,n-i-k} - \chi_{k+1,n-i-k} \quad (2 \leq i \leq n - 1, \ n \geq 4)
\end{align*}
\]

where
\[
g_{m,n}(t) := \prod_{k=m}^{n} \tilde{g}_k(t), \quad \tilde{g}_{m,n}(t) := \frac{1}{g_{m,n}(t)}
\]
(10)

and \( g_n = g_{n,n}, \tilde{g}_n = \tilde{g}_{n-n} \). Further, \( a, q \) and \( F \) are given by the following form:

For \( n = 2 \)
\[
a(y, \eta, t) = \chi_{2,1} y + g_{1,2} \eta_2 \\
q(y, \eta_2, t) = \chi_{2,2} y - \lambda_{1} \eta_2 \\
F(y, \eta_2, t) = \tilde{g}_{2,1} - \chi_{2,1} \tilde{g}_{1,2} f_1
\]
(11)
(12)
(13)

For \( n \geq 3 \)
\[
a(y, \eta, t) = \chi_{2,2} y + g_{1,2} \eta_2 \\
q(y, \eta, t) = \chi_{1,1} y + T \eta
\]
(14)
(15)

For the obtained virtual system, it is easy to confirm that \( q(y, \eta, t) \) and \( a(y, \eta, t) \) are bounded with respect to time \( t \) and Lipschitz with respect to \( y, \eta \) so that there exist positive constants \( L_1 \) and \( L_2 \) such that
\[
\begin{align*}
    ||q(y_1, \eta_1) - q(y_2, \eta_2)|| &\leq L_1 (||y_1 - y_2|| + ||\eta_1 - \eta_2||) \\
    ||a(y_1, \eta_1) - a(y_2, \eta_2)|| &\leq L_2 (||y_1 - y_2|| + ||\eta_1 - \eta_2||)
\end{align*}
\]
(18)
(19)

Further, the uncertain vector function \( F(y, \eta, t) \) can be evaluated from assumption (A-1) by
\[
||F(y, \eta, t)|| \leq \sum_{i=1}^{M_2} p_i |\phi_i(y)| + p_0
\]
(20)
with unknown positive constants $p_1$ and $p_0$ and a known smooth function $\phi_i(y)$ which has the following property for any variable $y_1$ and $y_2$:

$$|\phi_i(y_1 + y_2)| \leq |\phi_i(y_1)| |y_1| + |\phi_i(y_2)|$$  \hfill (21)

with a known smooth function $\phi_1$ and a function $\phi_2$ which is bounded for all bounded $y_2$.

The obtained virtual controlled system (6) has the relative degree of 1 and the zero-dynamics given by (12) or (15), which is obtained by a nominal system with $f_1$ and $F$ as disturbances, is exponentially stable so that from the converse theorem of Lyapunov [5], there exists a positive definite function $W(\eta)$ and positive constants $\kappa_1$ to $\kappa_4$ such that

$$\frac{\partial W(\eta)}{\partial \eta} q(0, \eta) \leq -\kappa_1 ||\eta(t)||^2, \quad ||\frac{\partial W(\eta)}{\partial \eta}|| \leq \kappa_2 ||\eta(t)||$$

$$\kappa_4 ||\eta(t)||^2 \leq ||W(\eta)|| \leq \kappa_3 ||\eta(t)||^2$$  \hfill (22)

### 3.2 Controller Design

We design a virtual input for the virtual system (6) through a robust adaptive output feedback control. The actual control input will be designed through the backstepping procedure as follows:

**Step 1:** Defining $\nu(t) = y(t) - y^*(t)$ as the tracking error, the virtual controlled system (6) can be rewritten as the following error system

$$\dot{\nu}(t) = a(\nu + y^*, \eta, t) + g_{1,n}u_{f1} + f_1(\nu + y^*, \eta, t) - y^*$$

$$\eta(t) = q(\nu + y^*, \eta, t) + F(\nu + y^*, \eta, t)$$  \hfill (23)

We design a virtual control input $\alpha_1$ for the input $u_{f1}$ in this error system (23) by

$$\alpha_1(t) = -[k(t)\nu(t) + u_R(t)]$$

$$k(t) = k_1(t) + k_P(t)$$

$$\dot{k}_1(t) = g_1(\nu(t))^2 - \sigma_1 k_1(t), \quad k_1(0) \geq 0$$

$$k_P(t) = \sum_{i=1}^{M_2} \gamma_1 i_1(\nu, y^*)^2 \nu(t)^2$$

$$u_R(t) = \gamma_1 \psi_1(y)^2 \nu(t)$$  \hfill (24-28)

where $\gamma_1$, $i_1$, $\gamma_1$ and $\sigma_1$ are arbitrary positive constants. Here consider a positive definite function $V_0(\nu, \eta, k)$:

$$V_0 = \mu_0 W(\eta) + \frac{1}{2} \mu_1 \nu(t)^2 + \frac{g_m}{2\gamma_1} [k_1(t) - k^*]^2$$  \hfill (29)

where $\mu_0$ and $\mu_1$ are any positive constants and $k^*$ is an ideal feedback gain to be determined later. $g_m$ is a positive constant such that $0 < g_m \leq g_{1,n}$.

The time derivative of $V_0(\nu, \eta, k)$ along the trajectories (23) and (26) yields that

$$\frac{dV_0}{dt} = \mu_0 \frac{\partial W(\eta)}{\partial \eta} [q(\nu + y^*, \eta, t) + F(\nu + y^*, \eta, t)] + \mu_1 \nu [a(\nu + y^*, \eta, t) + f_1(\nu + y^*, \eta, t) - y^*] + \mu_1 \nu [-g_1(\nu) + \nu_R(t, \nu + y^*)] + \mu_1 g_{1,n} \nu(u_{f1} - \alpha_1) + \mu_1 g_{1,n} \nu u_1 + R_0$$

$$+ \mu_1 \frac{g_m}{\gamma_1} [k_1 - k^*] [\gamma_1 \nu^2 - \sigma_1 k_1]$$  \hfill (30)

It follows from assumptions (A-1), (A-2) and (18) to (22) that we can evaluate the time derivative of $V_0$ by

$$\frac{dV_0}{dt} \leq -\mu_0 \kappa_1 ||\eta||^2 + \mu_0 \kappa_2 ||\eta|| L_1(\nu + \beta_0)$$

$$+ \mu_0 \kappa_2 ||\eta|| \left[ \sum_{i=1}^{M_2} \gamma_1 [\phi_i(\nu, y^*)] ||\nu|| + |\phi_2(\nu)|| \right]$$

$$+ \mu_0 \kappa_2 ||\eta|| [\rho_0 + L_2(\nu) + |y^*| + ||\eta||] ||\nu||$$

$$+ \mu_1 \nu d_1 [\nu_1(\nu + y^*)] + \mu_1 d_0 [\nu_1 + \nu_1(\nu + y^*)]$$

$$- \mu_1 g_{1,n} k_{\nu^2} - \mu_1 g_m k_{\nu^2} + \mu_1 g_m k_1$$

$$+ \mu_1 g_1(\nu) \psi_1(\nu + y^*)^2 - \mu_1 g_{1,n} g_{1,n} \nu(u_{f1} - \alpha_1) - \mu_1 \frac{g_m}{\gamma_1} [k_1 - k^*]^2$$  \hfill (31)

Further since we have

$$-\mu_1 g_{1,n} k_{\nu^2} + \mu_1 g_m k_{\nu^2} \leq -\mu_1 g_m k_{\nu^2}$$

from the fact that $k(t) \geq 0$ obtained from (26) and (27), and since we also have

$$-\mu_1 g_{1,n} g_{1,n} \nu(u_{f1} - \alpha_1)$$

$$\leq -\mu_1 g_m g_{1,n} \nu^2$$

we obtain from (31) that

$$\frac{dV_0}{dt} \leq -\left[ \mu_0 \kappa_1 - \rho_1 - \rho_2 - \sum_{i=1}^{M_2} \rho_3 \right] ||\eta||^2$$

$$- \left[ \mu_1 g_{1,n} k_{\nu^2} - \rho_0 \right] ||\nu||^2 + \mu_1 g_{1,n} \nu(u_{f1} - \alpha_1)$$

$$+ \frac{\mu_1}{4 \rho_4} [L_2 \rho_0 + \rho_0 + \rho_1 + \rho_2 + \frac{1}{4 \rho_0} \frac{\mu_1 g_m g_{1,n}}{\gamma_1} k_{\nu^2}^2]$$

$$- \mu_1 g_m \frac{\gamma_1}{\gamma_1} \left( 1 - \frac{\rho_5}{\gamma_1} \right) [k_1 - k^*]^2 + \mu_1 g_{1,n} g_{1,n} \nu^2$$

$$+ \frac{1}{4 \rho_2} \left[ L_1 \rho_0 + \rho_2 + \sum_{i=1}^{M_2} \rho_i |\phi_i(\nu)| + P_0 \right]$$

$$+ \sum_{i=1}^{M_2} \frac{1}{4 \rho_4} [\frac{\mu_1 g_{1,n} k_{\nu^2}}{\rho_3}]^2$$  \hfill (34)

with any positive constants $\rho_1$ to $\rho_5$ and

$$k_0 = \mu_1 L_2 + \rho_4 + \frac{1}{4 \rho_1} \left( \mu_0 \kappa_2 L_1 + \mu_1 L_2 \right)^2$$  \hfill (35)

Finally the time derivative of $V_0$ can be evaluated by

$$\frac{dV_0}{dt} \leq -\left[ \mu_0 \kappa_1 - \rho_1 - \rho_2 - \sum_{i=1}^{M_2} \rho_3 \right] ||\eta||^2$$

$$- \left[ \mu_1 g_{1,n} k_{\nu^2} - \rho_0 \right] ||\nu||^2 + \mu_1 g_{1,n} g_{1,n} \nu(u_{f1} - \alpha_1)$$

$$+ \mu_1 g_{1,n} \nu(u_{f1} - \alpha_1) + R_0$$  \hfill (36)
where \( \omega_1 = u_{f1} - \alpha_1, \rho_i = \frac{\phi_2 M \sigma_i}{\mu_{10} \gamma_i}, \) and

\[
R_0 = \frac{\mu_1}{4gM \gamma R} \frac{\mu_2^2}{4p_4} \left( L_2 \beta_0 + d_0 + \beta_1 \right)^2 + \frac{\left( \mu_1 q_2 \right)^2}{4p_2} \left[ L_1 \beta_0 + \sum_{i=1}^{M_2} g_i \phi_2 M + g_0 \right]
+ \frac{\mu_1 gM \gamma_i K^2}{4p_5 \gamma_i} + \sum_{i=1}^{M_2} \frac{1}{4\mu_1 gM \gamma_i} \left[ \frac{\left( \mu_1 q_2 p_i \right)^2}{4p_2} \right]^2.
\]

\( \phi_2 M \) in \( R_0 \) is a positive constant such that \( |\phi_2 M(\gamma^*)| \leq \phi_2 M \). Since \( \gamma^* \) is bounded, such a constant exists from the assumption that \( \phi_2 M(\gamma^*) \) is bounded for all bounded \( \gamma^* \).

**Step 2:** In step 2, we consider an error system, \( \omega^* \)-system, between \( u_{f1} \) and \( \alpha_1 \). \( \omega^* \)-system is given from (5) by

\[
\dot{\omega} = -\lambda_1 u_{f1} + u_{f2} - \alpha_1
\]

where

\[
\dot{\alpha}_1 = \frac{\partial \alpha_1}{\partial \gamma} \frac{\partial \gamma}{\partial \gamma} y^* + \frac{\partial \alpha_1}{\partial k_1} k_1
- \frac{\partial \alpha_1}{\partial \gamma} \left[ x_2,2y + g_1,1 \gamma_2 + f_1 + g_1,1 u_{f1} \right]
- \frac{\partial \alpha_1}{\partial \gamma} \left( \gamma \gamma \nu^2 - \sigma_1 k_1 \right)
\]

For this \( \omega^* \)-system, we design the virtual input \( \alpha_2 \) for \( u_{f2} \) as follows:

\[
\alpha_2 = -c_1 \omega_1 + \lambda_1 u_{f1} - \epsilon_1 \nu \omega_1 - \epsilon_1,1 \left( \frac{\partial \alpha_1}{\partial \gamma} \right)^2 y^2 \omega_1
- \epsilon_1,2 \left( \frac{\partial \alpha_1}{\partial \gamma} \right)^2 \omega_1 - \epsilon_1,3 \left( \frac{\partial \alpha_1}{\partial \gamma} \right)^2 \omega_2
- \epsilon_1,4 \left( \frac{\partial \alpha_1}{\partial \gamma} \right)^2 \omega_1 - \epsilon_1,5 \left( \frac{\partial \alpha_1}{\partial \gamma} \right)^2 \omega_1
- \epsilon_1,6 \left( \frac{\partial \alpha_1}{\partial \gamma} \right)^2 \omega_1 + \frac{\partial \alpha_1}{\partial k_1} \left( \gamma \gamma \nu^2 - \sigma_1 k_1 \right)
\]

where \( c_1, \epsilon_1,1 \) and \( \epsilon_1,1,6 \) are any positive constants.

Consider the following positive definite function \( V_1 \) for the obtained \( \omega^* \)-system.

\[
V_1 = \frac{1}{2} \omega_1^2 + V_0
\]

The time derivative of \( V_1 \) is obtained by

\[
\dot{V}_1 = \omega_1 \left( u_{f1} - \alpha_1 \right) + \dot{V}_0
\]

where \( \omega_2 = u_{f2} - \alpha_2 \).

Since \( g_1,1,2 \) and \( x_2,2 \) are bounded from assumption (A-2), we can evaluate the time derivative of \( V_1 \) by applying \( \alpha_2 \) given in (39) as follows:

\[
\dot{V}_1 \leq -c_1 \omega_1^2 + \omega_1 \omega_2 + \frac{\beta_i}{4\epsilon_1,2} \left( |\gamma|^2 + R_1 \right)
- \left[ \mu_1 q_2 \rho_1 - \rho_2 - \sum_{i=1}^{M} \rho_1 \right] ||\gamma||^2
- \left[ \mu_1 gM \sigma_{1,2} (1 - \rho_1) [k_1 - k^*]^2 + R_0 \right.
\]

with positive constants \( gM \) and \( \chi_M \) such that \( g_1,1,2 \leq gM \) and \( x_2,2 \leq \chi_M \) and

\[
R_1 = \frac{\mu_1 gM \gamma_i K^2}{4\epsilon_1,1} + \frac{\chi_2 \mu_1 q_2}{4\epsilon_1,2} + \frac{\chi_2 \mu_1 q_2}{4\epsilon_1,4} + \frac{\chi_2 \mu_1 q_2}{4\epsilon_1,5} + \frac{\beta_i}{4\epsilon_1,6}
\]

**Step 2** (3 \( \leq i \leq n - 1 \)): In step 2, we consider the error system, \( \omega_{i-1} \)-system, between \( u_{f1} \) and \( \alpha_1 \). \( \omega_{i-1} \)-system is given from (5) by

\[
\dot{\omega}_{i-1} = -\lambda_{i-1} u_{f1} - u_{f1} - \alpha_{i-1}
\]

where

\[
\dot{\alpha}_{i-1} = \frac{\partial \alpha_{i-1}}{\partial \gamma} \frac{\partial \gamma}{\partial \gamma} y^* + \frac{\partial \alpha_{i-1}}{\partial k_1} k_1
+ \frac{\partial \alpha_{i-1}}{\partial \gamma} \left( \gamma \gamma \nu^2 - \sigma_i k_1 \right)
\]

For this \( \omega_{i-1} \)-system, the virtual input \( \alpha_i \) for \( u_{f1} \) is designed by

\[
\alpha_i = -c_{i-1} \omega_{i-1} - \omega_{i-2}
+ \lambda_{i-1} u_{f1} - \epsilon_{i-1,1} \left( \frac{\partial \alpha_{i-1}}{\partial \gamma} \right)^2 y^2 \omega_{i-1}
- \epsilon_{i-1,2} \left( \frac{\partial \alpha_{i-1}}{\partial \gamma} \right)^2 \omega_{i-1} - \epsilon_{i-1,3} \left( \frac{\partial \alpha_{i-1}}{\partial \gamma} \right)^2 \omega_{i-1}
- \epsilon_{i-1,4} \left( \frac{\partial \alpha_{i-1}}{\partial \gamma} \right)^2 \omega_{i-1} - \epsilon_{i-1,5} \left( \frac{\partial \alpha_{i-1}}{\partial \gamma} \right)^2 \omega_{i-1}
- \epsilon_{i-1,6} \left( \frac{\partial \alpha_{i-1}}{\partial \gamma} \right)^2 \omega_{i-1} - \partial \alpha_{i-1} \left( \gamma \gamma \nu^2 - \sigma_i k_1 \right)
+ \sum_{k=1}^{i-2} \partial \alpha_{i-1} \left( -\lambda_k u_{f1} + u_{f1} \right)
\]

where \( c_{i-1}, \epsilon_{i-1,1} \) to \( \epsilon_{i-1,6} \) are any positive constants. Here we consider the following positive definite function \( V_{i-1} \):

\[
V_{i-1} = \frac{1}{2} \omega_{i-1}^2 + V_{i-2}
\]

The time derivative of \( V_{i-1} \) is then given by

\[
\dot{V}_{i-1} = \omega_{i-1} \left[ -\lambda_{i-1} u_{f1} + \omega_{i-1} + \alpha_i - \partial \alpha_{i-1} \left( x_2,2 y + g_1,1 \gamma_2 \right)
+ f_1 + g_1,1 u_{f1} \right] + \partial \alpha_{i-1} \left( \gamma \gamma \nu^2 - \sigma_i k_1 \right)
+ \sum_{k=1}^{i-2} \partial \alpha_{i-1} \left( -\lambda_k u_{f1} + u_{f1} \right) + \dot{V}_{i-2}
\]

It follows from the structure of \( \alpha_i \) given in (45) that \( V_{i-1} \) can be evaluated as
where

\[ V_{n-1} \leq -\sum_{k=1}^{n-1} c_k \omega_k^2 + \omega_{n-1} \omega_1 + \sum_{k=1}^{n-1} \frac{g_k^2}{4 \epsilon_{k,2}} |\eta|_2^2 + \sum_{k=1}^{n-1} R_k \]

\[ -\left[ \mu_0 k_1 - \rho_1 - \rho_2 - \sum_{i=1}^{M_2} \rho_{3i} \right] ||\eta||^2 \]

\[ -\mu_1 g_m \frac{\sigma_I}{\gamma_I} (1 - \rho_1) [k_I - k^*]^2 + R_T \]  

\[ \text{Step n: This is the final step. In step n, we design the actual control input as follows:} \]

\[ U = a_n \]  

\[ \text{using } a_n \text{ given in (39) or (45).} \]

In this final step, we finally consider the following positive definite function \( V: \)

\[ V = V_{n-1} = \frac{1}{2} \omega_{n-1}^2 + V_{n-2} \]  

The time derivative of \( V \) can be evaluated by

\[ \dot{V} \leq -\sum_{k=1}^{n-1} c_k \omega_k^2 - \mu_1 g_m k^* - k_0 |\nu|^2 \]

\[ -\left[ \mu_0 k_1 - \rho_1 - \rho_2 - \sum_{i=1}^{M_2} \rho_{3i} - \sum_{k=1}^{n-1} \frac{g_k^2}{4 \epsilon_{k,2}} \right] ||\eta||^2 \]

\[ -\mu_1 g_m \frac{\sigma_I}{\gamma_I} (1 - \rho_1) [k_I - k^*]^2 + R_T \]  

\[ \text{using the same manner as in previous steps. Where } R_T = \sum_{k=1}^{n-1} R_k. \]

Setting \( \rho_1 = \rho_2 = \mu_0 k_1, \rho_{3i} = \frac{\mu_0 k_1}{10 M_2} \) and \( \rho_4 = \frac{1}{2} \) and considering \( \mu_0 \) such as \( \mu_0 \geq \frac{5(a-1)b k_1}{4 \epsilon_{k,2} \epsilon_{k,3}} \), we have

\[ \dot{V} \leq -\sum_{k=1}^{n-1} c_k \omega_k^2 - \frac{k_1}{2 \epsilon_k} \mu_0 W(\eta) - \mu_1 g_m k^* - k_0 |\nu|^2 \]

\[ -\mu_1 g_m \frac{\sigma_I}{\gamma_I} (1 - \rho_1) [k_I - k^*]^2 + R_T \]  

\[ \text{from the fact that } ||\eta||^2 \geq \frac{1}{\epsilon_k} W(\eta). \]  

Further by considering the ideal feedback gain \( k^* \) such that

\[ k^* \geq \frac{1}{\mu_1 g_m} \left[ \frac{k_1 \mu_1}{2 \epsilon_k} + k_0 \right] \]  

it follows that

\[ \dot{V} \leq -\sum_{k=1}^{n-1} c_k \omega_k^2 - \frac{k_1}{2 \epsilon_k} \left[ \mu_0 W(\eta) + \frac{1}{2} \mu_1 |\nu|^2 \right] \]

\[ -\frac{1}{2} \frac{\sigma_I}{\gamma_I} \left[ \mu_1 g_m k^* + \frac{1}{2} \mu_1 |\nu|^2 \right] + R_T \]

\[ \leq -\sum_{k=1}^{n-1} c_k \omega_k^2 - c_m V_0 + R_T \]  

where \( c_m = \min \left[ \frac{\mu_1 g_m}{\epsilon_k}, \sigma_I \right]. \)

Finally, the time derivative of \( V \) can be evaluated by

\[ \dot{V} \leq -\alpha_v V + R_T \]  

where \( \alpha_v = \min [2c_1, \ldots, c_{n-1}, c_m]. \)

It is apparent from (56) that all the signals in the closed-loop system with the controller (50) are bounded. We also obtain from (56) that

\[ \lim_{t \to \infty} V \leq R_T/\alpha_v \]

From the fact that \( |\nu|^2 \leq 2V/\mu_1 \), it follows that

\[ \lim_{t \to \infty} |\nu|^2 \leq 2R_T/\alpha_v \mu_1 \]

Thus the control objective (4) is achieved for \( \delta \) such that \( \delta^2 \geq 2R_T/\alpha_v \mu_1 \)

and it is also to easy to confirm that an appropriate choice of \( \mu_0, \mu_1 \) and \( \rho_4 \) and design parameters \( \gamma_I, \gamma_I, \gamma_R, \epsilon_{10} \) and \( \epsilon_{i,1} \) to \( \epsilon_{i,6}(1 \leq i \leq n-1) \) ensure the control objective for any positive constant \( \delta \). As a conclusion we have the following theorem.

**Theorem:** Under assumptions (A-1) and (A-2) on the controlled system (1), all the signals in the resulting closed-loop system with controller (50) designed according to each step with adaptive adjusting law (25) to (27) and the robust control term (28) are bounded and there exist appropriate design parameters \( \gamma_I, \gamma_I, \gamma_R, \epsilon_{10} \) and \( \epsilon_{i,1} \) such that the tracking error \( \nu(t) \) converges to any given bound \( |\nu(t)| \leq \delta \) as \( t \to \infty \).

4 NUMERICAL SIMULATION

Here the effectiveness of the proposed method will be confirmed through a numerical simulation for a model of a DC motor. The model considered here is given as follows:

\[ x_1 = -F(x_1) + K_T x_2 - T(t) \]

\[ x_2 = -\frac{K_T}{L(t)} x_1 - \frac{R(t)}{L(t)} x_2 + \frac{1}{L(t)} u \]

\[ y = x_1 \]  

\[ x_1 \] is the rotational velocity of the motor and \( x_2 \) is the armature current. \( R(t) \) and \( L(t) \) are the resistance of armature winding and the self-inductance, respectively, which are assumed to be time-varying related to temperature in the motor. \( K_b, K_t, T(t) \) and \( J \) are the back-emf parameter, the torque motor parameter, the load torque and the rotor inertia, respectively. \( F(x_1) \) is the friction given in the following form [7].

\[ F(x_1) = F_0(x_1) + \theta x_1 \]

\[ F_0(x_1) = F_c sgn(x_1) + (F_s - F_c) \exp \left\{ \left( \frac{x_1}{v_s} \right)^2 \right\} sgn(x_1). \]
Table 1: Motor and friction model parameters.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Values</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>J</td>
<td>0.2</td>
<td>kgm²</td>
</tr>
<tr>
<td>K₁</td>
<td>0.306</td>
<td>Nm/A</td>
</tr>
<tr>
<td>K₂</td>
<td>3.15</td>
<td>V/s/rad</td>
</tr>
<tr>
<td>θ</td>
<td>0.4</td>
<td>rad/m</td>
</tr>
<tr>
<td>vₙ</td>
<td>0.001</td>
<td>rad/s</td>
</tr>
<tr>
<td>F₀</td>
<td>1</td>
<td>Nm</td>
</tr>
<tr>
<td>Fᵣ</td>
<td>1.5</td>
<td>Nm</td>
</tr>
</tbody>
</table>

The parameters of the DC motor and the friction model are shown in Table 1. In this simulation we assumed that R(t) and L(t) are varied as shown in Fig. 1 and T(t) is given as shown in Fig. 2.

In order to apply the proposed method, we consider the following nonsingular transformation:

\[ z_1 = x_1 \]
\[ z_2 = \frac{J R(t)}{K L(t)} x_1 + x_2 \]

so that the system (59) can be represented by

\[ \dot{z}_1 = f_1(z_1, t) + g_1 z_2 \]
\[ \dot{z}_2 = f_2(z_1, t) + g_2(t) u \]
\[ y = z_1 \]

where

\[ f_1(z_1, t) = -\frac{1}{J} \left[ R_0(z_1) + \theta z_1 + \frac{J R(t)}{L(t)} z_1 \right] - \frac{T(t)}{J} \]
\[ f_2(z_1, t) = -\frac{R(t)}{K L(t)} R_0(z_1) - \frac{T(t) R(t)}{K L(t)} \]
\[ + \frac{J}{K} \frac{d}{dt} \left( \frac{R(t)}{L(t)} \right) - \frac{\theta R(t)}{K L(t)} - \frac{K_0}{L(t)} \]
\[ g_1 = \frac{K_0}{J}, \quad g_2(t) = -\frac{1}{L(t)} \]

Since the transformed system (62) satisfies the assumptions (A-1) and (A-2) with known functions as \( \psi_1(y) = \psi_2(y) = y \), we can design the controller for the DC motor according to the proposed procedure.

Figs 3 and 4 show the simulation results. A good control result is obtained in spite of the controlled system having unknown nonlinearities and time-varying unknown coefficients in control input term.

5 CONCLUSIONS

In this paper, we proposed a robust adaptive tracking control, which is based on high-gain feedback based adaptive control, for a class of uncertain time-varying nonlinear systems with higher order relative degree. The effectiveness of the proposed method was confirmed through a numerical simulation for a model of a DC motor.

References